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Peak Extraction in Daylight Simulations Using BDSF Data

David Geisler-Moroder¹, Gregory J. Ward², Taoning Wang³, Eleanor S. Lee³

¹Bartenbach GmbH, Aldrans, Austria

²Anywhere Software, Berkeley, California, USA

³Lawrence Berkeley National Laboratory, Berkeley, California, USA

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Abstract

Bidirectional scattering distribution function (BSDF) data are used in design practice to represent optically complex daylighting and solar control systems in lighting and energy simulation software. Visual comfort assessments (e.g., daylight glare) require accurate determination of luminance and corresponding solid angle of glare sources. For the sun, the necessary resolution of the BSDF causes problems both in terms of data volume and computational effort. With “peak extraction” (PE), we present a new method that simulates the direct solar contribution at its real size and spread, while efficiently using the underlying BSDF data set for the scattered light. PE enables practitioners to evaluate daylight performance metrics for their designs at improved accuracy.

Key Innovations

- Novel peak extraction method to extract direct transmission from daylighting and solar control system BSDF data
- Improved accuracy of glare calculations (e.g., DGP) and daylight renderings (shadow patterns)

Practical Implications

The PE algorithm presented in this study provides practitioners with a method of simulating daylighting systems at higher accuracy and efficiency. It is suited for systems with a view component such as typical fabrics with some openness which are widely used as glare protection devices. It can also be applied for Venetian blinds, but the use of proxy geometry is preferred here to allow also the characteristic striped shadow pattern. The method should not be applied to systems that do not have a direct, “see-through” component.

Introduction

Over the past years, tools and processes have been developed to accurately characterize and simulate the performance of optically complex daylighting and solar control systems in buildings using BSDF data (Nicodemus et al., 1977; IEA, 1999; Ward et al., 2011). BSDFs describe how light from each incident direction is scattered (reflected and transmitted) by a simple or composite surface, such as a window shade. BRDFs (R

for reflectance) describe surface properties of opaque materials (that e.g. can make up a shading system such as slats of a Venetian blind). BTDFs (T for transmittance) describe properties of transmissive materials and systems (e.g., translucent panels or acid etched lamellas). Both reflectance and transmittance scattering properties are included in BSDFs.

BSDF data can be generated using parametric or data-driven models. Parametric or analytical BSDF models are widely used in computer graphics. As examples, a clear glazing or an ideal mirror are described by Dirac delta functions defining the transmittance or reflectance for the direct transmitted or mirrored direction, respectively, and zero elsewhere. Examples for scattering models include the Phong model (Phong, 1975), Cook-Torrance model (Cook and Torrance, 1981), and Ward-Geisler-Moroder-Dür model (Geisler-Moroder and Dür, 2010). These analytical models are widely used for generic material descriptions in simulations, but their assumptions must be reviewed if applied to façade systems that differ from those initially considered. Data-driven models are based on measured angularly-resolved data of real-world materials and systems. A detailed description on how to generate BSDFs from these measurement data is given by Ward et al. (2014), Lee et al. (2018), and Geisler-Moroder and Lee (2021).

Tabulated BSDFs are derived from BSDF models, i.e., BSDF data described by a discrete set of values for a defined number and set of directions (Ward et al., 2021). Various angular basis representations for tabulated BSDFs have been defined for different simulation purposes (Geisler-Moroder and Lee, 2021). The resolution of tabulated BSDF data needs to match the optical properties of the represented system and the respective application.

While low resolution BSDF data is likely sufficient for calculations of daylight autonomy based on hourly illuminance, high resolution BSDF data are needed to represent the direct solar component for calculations of discomfort glare and other metrics requiring granular spatial modeling of sunlight, especially for systems that allow specular transmission or reflection, e.g., for fabrics with openness, blinds, or (mirror) louvers (Ward and McNeil, 2011; McNeil, 2011; Ward et al., 2012; Geisler-Moroder et al., 2017; Lee et al., 2018; Grobe, 2019).

Sunlight, whether transmitted, scattered, or reflected, needs to be predicted at highest accuracy due to its high intensity. An accurate representation of the direct solar contribution is critically important when it comes to luminance-based output. This includes evaluation of visual comfort (e.g., using the daylight glare probability (DGP) metric (Wienold et al., 2019)) as well as realistic appearance of physically-based renderings (e.g., sun in the field of view or sharp shadow patterns). Results from sensitivity analyses (Pedersen and Rasmussen, 2019; Geisler-Moroder, 2019) indicate that high-resolution BSDFs with a minimum basis resolution of 4096 x 4096 are required for accurate glare evaluations. Use of high-resolution BSDFs, however, tests the computational limits of Monte Carlo sampling in a backward raytracing approach to adequately resolve the peaks in the distribution.

As an option for detailed calculations of the direct solar transmitted or reflected component, proxy geometry can be used for the direct component simulation in the shadow testing algorithm (Ward, 2011). As an example, for venetian blinds, this enables the rendering of shadow patterns instead of an averaged, extenuated light patch inside the room. With this, the direct solar contribution to interior light levels can be calculated with high accuracy. Additionally, proxy geometry provides a correct appearance of the system in the façade. However, the provision of exact system geometry is not always possible, either because it is not available (e.g., for fabrics or expanded metal mesh) or prohibited because of intellectual property (IP) protections (e.g., for blind systems).

Nominally, stratified importance sampling sends multiple rays in directions corresponding to the BSDF. We may also send rays specifically towards known light sources using the BSDF to scale the transmission or reflection. Both of these methods tend to spread out peaks in the BSDF. Ray-tracing methods in general, whether forward (e.g., photon-mapping) or backward (e.g., Whitted-style) rely on these sampling strategies to reduce variance and improve accuracy in the results (Schregle, 2004). The strategy we are missing with tabulated BSDF sampling is following a single ray in the direction of specular transmission. Since tabulated BSDFs are designed to handle the more general problem of integrating arbitrary distributions, they do not employ single-ray specular sampling, thus missing out on the most efficient strategy for which ray-tracing was originally introduced (Whitted, 1980).

This prompted us to develop a new method to separate the direct solar component transmitted through a façade system represented via its tabulated BSDF data set.

Method

For small but bright light sources (e.g., the sun), RADIANCE complements the stochastic backwards raytracing algorithm with a deterministic component (Ward and Shakespeare, 1998). In order to apply this approach also for the direct component passing through a daylighting system, a peak extraction (PE) algorithm

was introduced that analyzes the BSDF data during the raytracing process. Thereby it is decided if a direct, deterministic component is simulated or not.

In theory, a “direct through” component is a delta peak in the BSDF. Every representation with finite resolution averages the peak (e.g., direct solar contribution) over a defined solid angle. While the transmitted flux is correct, the resulting peak luminance can be reduced by orders of magnitude with low-resolution BSDFs since the light flux is spread over a bigger area. I.e., the degree of scattering depends on the angular resolution of the BSDF bases. This is of high relevance for luminance-based calculations, such as glare evaluations, and for realistic renderings where correct shadow patterns are desired.

The newly developed PE algorithm analyzes the tabulated BSDF data for every ray during Monte Carlo sampling that hits the respective daylighting or shading system surface and determines whether the underlying distribution has a peak in the tested direction. By checking surrounding directions (see Figure 1), the algorithm determines if there is a strong local peak in the distribution. If so, the peak is replaced with a direct specular component where the transmission is calculated from the local BSDF value.

While relatively simple in its approach, the details of assigning flux correctly during PE are involved, requiring careful tuning to satisfy energy conservation and rendering criteria. Whenever a transmitted specular “direct through” component in the unscattered direction is identified, it is assigned an integrated transmission value equal to the BSDF value times the solid angle of the peak patch. The calculation is then adapted such that: (i) any “shadow ray” sent towards a light source and striking the BSDF material will pass directly through, modified by the transmission value in this direction computed from the BSDF, and assigned a solid angle equal to that of the associated source object, which is 0.533° in the case of the sun disc; and (ii) any “view ray” sent directly from the virtual camera (i.e. corresponding to line-of-sight) is transmitted unperturbed, again modified by the computed transmission value in the respective direction.

To avoid double-counting of source contributions, specular transmission from light sources near the “direct through” direction is muted. These rays are given an average brightness determined from the 28 surrounding test directions shown in Figure 1. Similarly, near-specular transmitted view rays are rejected to avoid over-estimation of scattering around the view direction.

The identification of the “direct through” component involves evaluating multiple locations around the unscattered direction and checking for conditions where a peak is present. Additional to the initial ray direction, the peak extraction algorithm evaluates 28 BSDF directions within 2.4 times the “search radius” (see Figure 1), i.e., the smallest angular resolution of the transmitted BSDF, where the search radius basis is one maximum resolution division (in respective direction) in

the associated representation. This allows for slight measurement or interpolation errors in the underlying BSRF data. The code tries to find BSRF values that (i) cover the smallest represented solid angle in this direction, and (ii) average more than 1.5 times as bright as the nearby, surrounding BSRF directions. If both conditions are met, then this peak BSRF is used to define the “through” component (specular transmittance) and to calculate the transmission value in this direction.

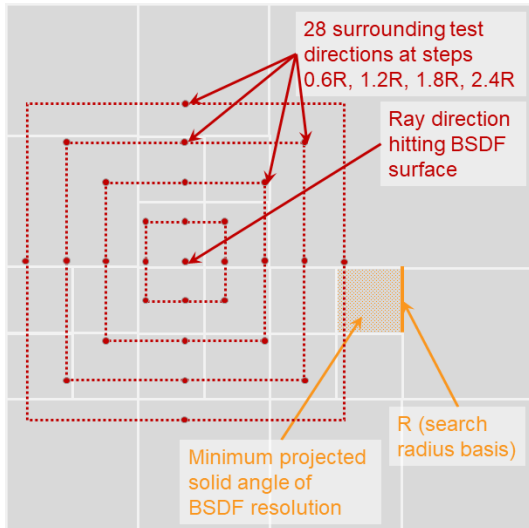


Figure 1: Example local BSRF tensor tree data structure (gray) with minimum projected solid angle and corresponding search radius (orange) and resulting test directions (red).

The search parameters and thresholds in the peak extraction algorithm (28 directions, 2.4 times search radius, and 1.5 times average value) were determined empirically through several experiments with real and synthetic BSRF data to give good performance under most conditions.

The experiments also show that there is an interaction between finding peaks in the BSRF data and the underlying resolution. For a clear glass with only unperturbed transmission all transmitted flux will end up in a single BSRF patch, independent of the basis resolution (see Figure 2, top row). In this case, the peak extraction algorithm will always work and extract the direct component. Looking at a synthetic example of a scattering translucent panel with full-width-tenth-max of 5° one can see that most of the flux is summarized in one patch when using a low resolution Klems BSRF (Figure 2, bottom left), while a high-resolution tensor tree representation (Figure 2, bottom right) distributes the flux into several patches according to the shape of the smooth peak. This gives a good example where the PE algorithm should not be applied since the translucent panel does not have a clear view component. I.e., PE would erroneously model specular transmission if a low resolution BSRF was used.

A similar effect occurs if the tabulated BSRF data set is derived from goniophotometer measurements. If a high-resolution tensor tree is used whose maximum resolution is as good or better than the measurement data, the

BSRF will resolve the angular spread of the goniophotometer rather than the daylight system being measured. This may prevent peak extraction from being triggered, since the peak will be spread into a larger region than expected, although there would be a view component to be extracted.

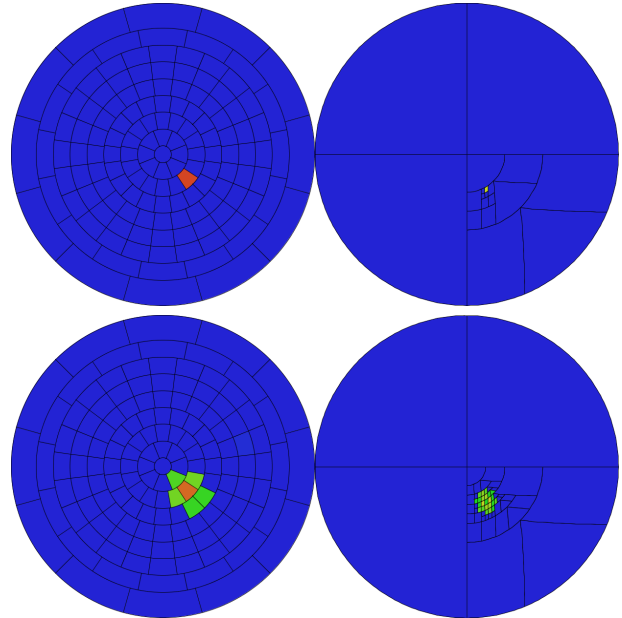


Figure 2: Example BSRF for specular glass (top row) and a scattering translucent panel with full-width-tenth-max of 5° (bottom row), each in Klems resolution (left) and variable tensor tree with maximum resolution of 4096x4096 (right).

Overall, PE enables accurate simulation of direct light contributions for metrics such as discomfort glare that require such resolution and for shading systems that allow specular transmission. This means that glare sources are rendered with their assumed real size (e.g., sun disc), independent of the BSRF resolution, and thus are correctly evaluated in widely used, luminance-based daylight glare metrics such as DGP and DGI. In turn, for systems with a see-through component, PE enables use of BSRF resolutions coarser than would be needed to resolve the 0.533° apex angle of the sun.

In the RADIANCE lighting simulation software (Ward and Shakespeare, 1998) the new method has been implemented in the “aBSDF” material. By assigning this material the user tells the software to look for a possible peak in direct transmission. The aBSDF material is implemented as part of the open-source RADIANCE code and can be viewed online (Radsite, 2021).

Figure 3 shows how to specify a BSRF and an aBSDF material in RADIANCE. Defining an aBSDF material is similar to the existing BSRF material with the only difference that no thickness of the system has to be defined (first argument after specification of number of

void BSRF my_BSRF	void aBSDF my_aBSDF
6 0 system.xml 0 0 1 .	5 system.xml 0 0 1 .
0	0
0	0

Figure 3: Example code to define a BSDF (left) and aBSDF (right) material in RADIANCE.

arguments). As the thickness is needed whenever proxy geometry is used, this argument is redundant for aBSDF. Then the XML file holding the BSDF data and the system’s “up-vector” (usually +Z when installed in the façade) is given. Finally, a transformation can be defined or – as in the example – skipped with a dot.

Results

An initial test shows the desired result of the novel PE algorithm. The simple test case in Figure 4 shows the benefit of using the PE algorithm in terms of the see-through component of a partially open daylighting system. The window is equipped with a fabric shade with 2% openness factor, using an isotropic variable resolution tensor tree data set with a maximum resolution of 4096x4096. The lower image with PE in Figure 4 clearly shows the differences in terms of an undistorted view through the perforated fabric (note the tree and horizon outside the window in the lower image), as well as the expected sharp shadow patterns indoors both at the wall and on the table.

In a second example – a typical office application – the same fabric shade as in the prior example with 2% openness factor was used. Here the simulations also show the difference in both the visual appearance of the

(“imperceptible” glare). BSDF resolution: max. 4096x4096.

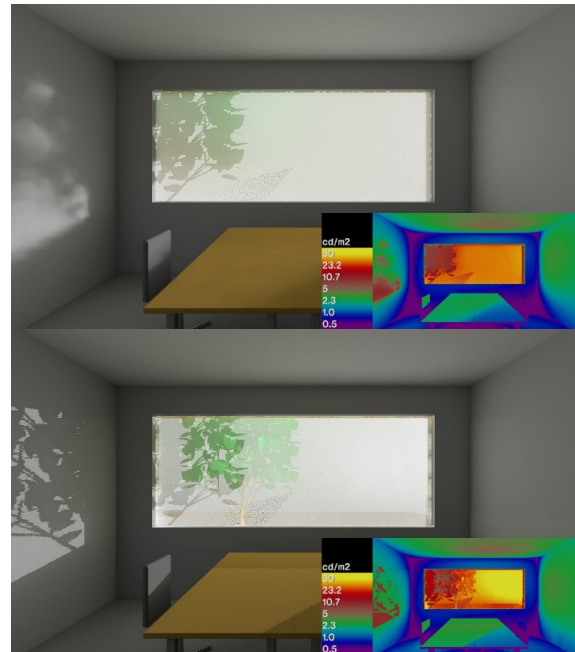


Figure 4: Simulation without peak extraction (top) and with peak extraction (bottom). Note the differences in the view to the outside and in the shadow patterns at the wall and the table. BSDF resolution: max. 4096x4096.

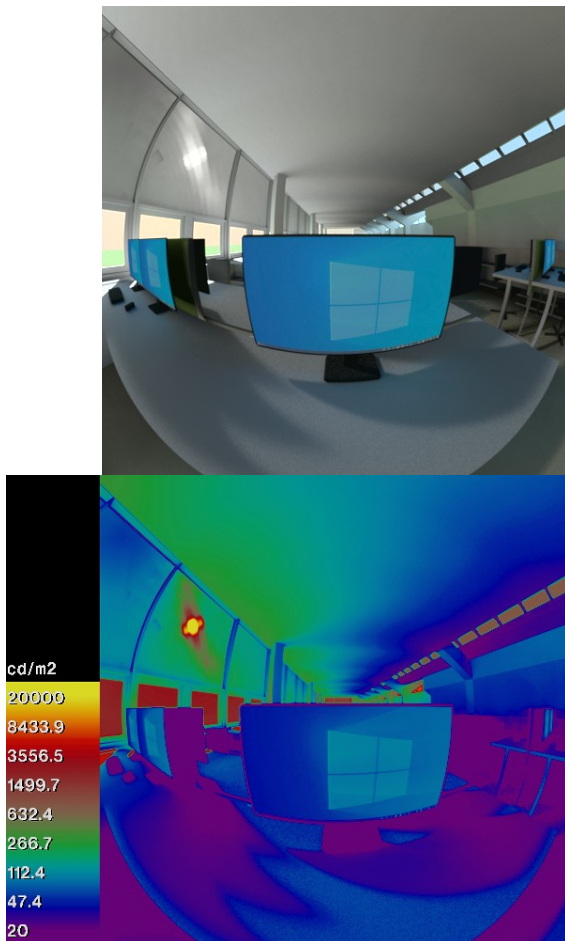


Figure 5: Simulation without PE. The maximum luminance (sunspot) is 163K cd/m², the DGP is 0.248

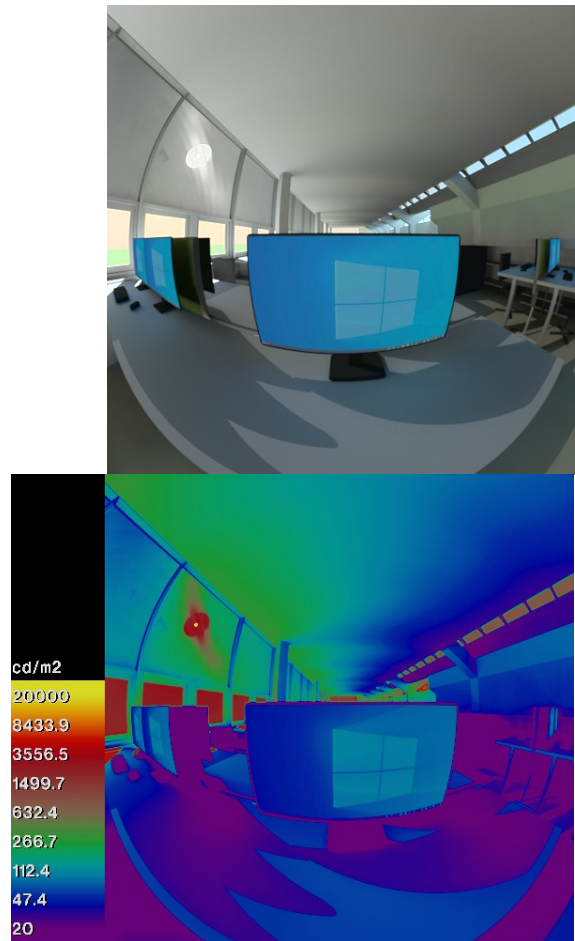


Figure 6: Simulation with PE. Note different shadow patterns and size of the solar disk. The maximum

luminance (sunspot) is 4,260K cd/m², the DGP is 0.359 (“noticeable” glare). BSDF resolution: max. 4096x4096.

shadow pattern in the room and the size and luminance of the direct sun in the field of view (Figures 5 and 6). The maximum luminance in this example changes from 163K cd/m² without PE (Figure 5) to 4,260K cd/m² with PE (Figure 6). The DGP value, a metric for daylight discomfort glare, increases from 0.248 (“imperceptible” glare) to 0.359 (“noticeable” glare).

In a third test, we show the effect of the PE algorithm for different tabulated BSDF resolutions. Figure 7 shows a simple box room with a direct view to the façade, which is equipped with the same fabric shade with 2% openness factor. The left column shows the results for using the BSDF without PE, the right column with applying the PE method. The results in the first row use a Klems BSDF with a resolution of 145x145, the second, third and fourth row use isotropic variable resolution tensor tree data sets with maximum resolutions of 1024x1024, 4096x4096, and 16384x16384, respectively. In all cases, the PE method extracts the “direct through” component and puts the light flux into the correct solid angle of the sun disc (see small yellow dot in close-up insert). From the 180° fisheye images in Figure 7 the vertical illuminances (Ev) and the DGP values are calculated (Table 1). While the illuminance values show a good match between the versions with and without the PE algorithm, the DGP results differ significantly. For the tensor tree BSDFs, without use of PE, the DGP increases from about 0.22 to 0.28 with increased resolution due to a smaller – and thus brighter – peak; both values predict “imperceptible” glare. When using PE, the DGP value of about 0.42 predicts “disturbing” glare, independent of the BSDF basis resolution used. The drop in Ev and DGP at the highest resolution is due to differences in transmission caused by sampling noise during the generation of the tabulated BSDFs. The increased computational effort for the highest resolution does not appear warranted for this modeled condition (insignificant change in DGP rating), however such resolution may be justified for other conditions or analysis objectives (e.g., prototyping). The lower Ev result for the Klems case is due to a lower predicted average transmission in the direction of the sun. This is caused by the fact that the sun is close to the cut-off angle of the fabric in this example and lies within a Klems patch where neighboring, shaded directions are averaged in. This shows that Klems resolution would not be adequate to represent the system accurately, but a tensor tree with a maximum resolution of 1024x1024 would suffice.

Table 1: Vertical illuminance and DGP values calculated from test room renderings in Figure 7 using different BSDF resolutions with and without PE.

BSDF resolution	Without PE		With PE	
	Ev	DGP	Ev	DGP
Klems:				
145x145	3881	0.19	3891	0.38
	x	1	x	9

Tensor tree:				
max. 1024x1024	4441	0.22	4331	0.41
	x	2	x	5
max. 4096x4096	4401	0.26	4391	0.41
	x	7	x	8
max. 16384x16384	4411	0.27	4301	0.40
	x	6	x	9

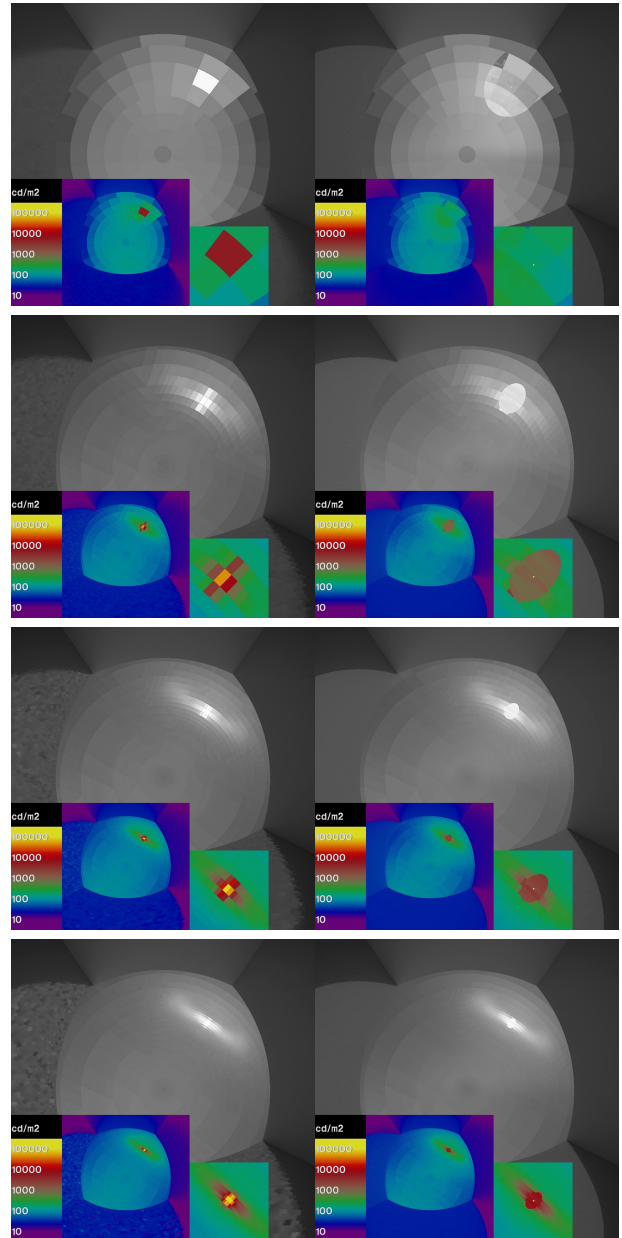


Figure 7: Test room renderings for Klems 145x145 (upper row) and tensor tree bases with resolution: 1024x1024 (second row), 4096x4096 (third row), and 16384x16384 (lower row) for a fabric shade at the façade with PE (right column) and without PE (left column).

Discussion

The current implementation of the PE algorithm is limited to direct-through contributions, which is often the most critical peak contribution for luminance-based daylight performance metrics, especially for glare

evaluations. However, if off-specular and upward-reflected peaks are expected, the method of photon mapping with high-resolution BSDFs (Grobe, 2019a) is recommended. Photon mapping adds the benefits of forward raytracing for small and high intensity light sources (e.g., the sun) to the general backward raytracing functionality and thus allows one to simulate reflected peaks efficiently and with reduced noise (Grobe, 2019b). However, with photon mapping, the sharpness and intensity of any directional contribution, whether directly transmitted or specularly reflected light, is limited by the resolution of the underlying BSDF data. Both contributions lead to blurry shadow edges or – when looking from indoors – to reduced peak luminance values.

As another option for detailed calculations of the solar component – directly transmitted or redirected – proxy geometry can be useful in various simulation methods but is still limited within the scope of possibilities of backward raytracing. BSDFs generated using genBSDF are compatible for PE if the tabulated BSDF is generated using sufficiently high parameters to model the intensity and angular distribution of peaks.

This also opens the topic for future work. As a next step, an adaptation and extension of the PE algorithm to also account for reflected peaks should be worked out. First ideas exist and include pre-processing of the overall BSDF data set to find peaks in the distribution independent of the incident direction.

First evaluations from a pilot validation study revealed the benefits of the PE method compared to using only the tabulated BSDF data for a specific anisotropic shade fabric (Ward et al., 2021). To further highlight the impact of the proposed method, extensive testing based on real-world application examples from daylighting design projects will be done. This will show the effects of the new method on different daylight performance metrics for both point-in-time and annual calculations.

Conclusion

We presented a novel peak extraction methodology to separate the direct solar component from light transmitted through a daylighting or solar control system with a view component when represented by a tabulated BSDF data set. This allows the evaluation of daylight performance metrics at higher accuracy and improved efficiency, especially for luminance-based evaluations such as glare calculations (e.g., DGP) and daylight renderings showing photorealistic shadow patterns (e.g., contrast glare and visual appearance).

The new method is applicable for all daylighting and solar control systems that are characterized by a view component, or by a direct-through transmission component. This includes many of the widely-used window attachments in buildings as e.g., glare protection fabrics with some openness fraction, blinds, perforated louvers, expanded metal meshes, or even clear glazing.

In summary, the new PE algorithm improves accuracy and quality of daylight simulations with BSDF data. As

it is difficult and costly to rectify problems associated with occupant discomfort after the purchase of capital equipment such as windows and shades, accurate simulations inform product selection and ultimately improve indoor environmental quality.

The PE methodology is an important step forward in enabling practitioners to correctly evaluate daylighting and visual comfort in buildings taking into account the planned or installed system technologies. For industry and manufacturers, PE can make it easier or even possible to generate and provide adequate BSDF data for some products in their portfolio. The data must be high enough resolution to properly capture the scattered light, but not to resolve the sun. The Klems case in Figure 7 shows that a too coarse resolution causes other problems. Further investigation will be required.

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